ADICHUNCHANAGIRI UNIVERSITY

18MAT31

Third Semester BE Degree Examination January 2020
(CBCS Scheme)

Time: 3 Hours | Max Marks: 100 marks

Sub: Engineering Mathematics - III

Q P Code: 60301

Instructions: 1. Answer **five full** questions.
2. Choose one full question from each module.
3. Your answer should be specific to the questions asked.
4. write the same question numbers as they appear in this question paper.
5. Write Legibly

Module – 1

1 a Find the Laplace transform of $\frac{\cos at - \cos bt}{t}$. — 7 marks

b A periodic function f(t) of period a, $a > 0$ is defined by — 6 marks

$$f(t) = \begin{cases} E & 0 < t < a/2 \\ -E & \frac{a}{2} < t < a \end{cases}$$ Show that $L[f(t)] = \frac{E}{s}\tanh\left(\frac{as}{4}\right)$. 2

c Solve the differential equation $\frac{d^2y}{dt^2} + 4\frac{dy}{dt} + 4y = e^{-t}$ with $y(0) = 0 = y'(0)$ by using Laplace transforms. — 7 marks

7

Or

2 a Find $L^{-1}[\cot^{-1}s]$. — 7 marks

b Express $f(t) = \begin{cases} \sin t, & 0 < t < \pi \\ \sin 2t, & \pi < t < 2\pi \\ \sin 3t, & t > 2\pi \end{cases}$ — 6 marks

in terms of unit step function and hence find their Laplace transform f (t).

c Using Convolution theorem obtain inverse transformation of $\frac{s^2}{(s^2+a^2)(s^2+b^2)}$. — 7 marks

Module – 2

3 a Find the Fourier series for the function $\frac{\pi - x}{2}$ in $0 < x < 2\pi$. 7 — 7 marks

Hence deduce that $\frac{\pi}{4} = 1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \ldots\ldots$

b Find half range sine series for $f(x) = \begin{cases} \frac{1}{4} - x & 0 < x < 1/2 \\ x - \frac{3}{4} & 1/2 < x < 1 \end{cases}$. 5 — 7 marks

c Express y as a Fourier series upto the first harmonic given. — 6 marks

x	0	π/3	2π/3	π	4π/3	5 π/3	2π
y	1.98	1.30	1.05	1.30	- 0.88	- 0.25	1.98

5

Or

4 a Obtain Fourier series for the function $f(x) = |x|$ in $-\pi \le x \le \pi$ — 7 marks

hence deduce that $\frac{\pi^2}{8} = \sum_{n=1}^{\infty} \frac{1}{(2n-1)^2}$.

b Expand $f(x) = 2x - 1$ as a cosine half range Fourier series in $0 < x < 1$. — 6 marks

PTO

c Obtain constant term and the coefficients of the first sine and cosine terms in the Fourier expansion of y from the table. 7 marks

x	0	1	2	3	4	5
f(x)	9	18	24	28	26	20

Module – 3

5 a Find the Fourier sine transforms of $f(x) = \frac{1}{x(1+x^2)}$. 7 marks

b Find the Fourier cosine transform of $f(x) = \frac{1}{1+x^2}$. 6 marks

c Solve the difference equation $y_{n+2} + 6y_{n+1} + 9y_n = 2^n$, with $y_o = y_1 = 0$ by using z transform. 7 marks

Or

6 a Find the Fourier sine transform of $e^{-|x|}$ Hence show that $\int_0^\infty \frac{x\sin mx}{1+x^2}\,dx = \frac{\pi}{2}e^{-m}$, $m > 0$. 7 marks

b Obtain the Z-transform of $2n + \sin(n\pi/4) + 1$. 6 marks

c Obtain the inverse Z-transform of $\frac{2z^2+3z}{(z+2)(z-4)}$. 7 marks

Module – 4

7 a Employ Taylor's method to find y at x=0.1 and 0.2 correct to four places of decimal in step size of 0.1 given the linear differential equation $\frac{dy}{dx} - 2y = 3e^x$ whose solution pasess through the origin. 7 marks

b Using fourth order Runge – Kutta method to find y at x = 0.1 given that $\frac{dy}{dx} = 3e^x + 2y$, $y(0) = 0$, taking h = 0.1. 7 marks

c Given that $\frac{dy}{dx} = x - y^2$ and the data y(0) = 0, y(0.2) = 0.02, y(0.4) = 0.0795 , Y(0.6) = 0.1762 ,find y(0.8) by using Adam- Bashforth method. 6 marks

Or

8 a Using modified Euler's method find y(20.2) and y(20.4) given that $\frac{dy}{dx} = \log_{10}\left(\frac{x}{y}\right)$ with y(20) = 5 taking h=0.2. 7 marks

b Solve : $(y^2 - x^2)dx = (y^2 + x^2)dy$ for x = 0 (0.2) 0.4 given that y = 1 at x = 0 initially, by applying Runge-Kutta Method of order 4. 7 marks

c Apply Milne's Predictor and Corrector formulae to compute y(1.4) correct to four decimal places. 6 marks

given $\frac{dy}{dx} = x^2 + \frac{y}{2}$ with

x	1	1.1	1.2	1.3
y	2	2.2156	2.4649	2.7514

Module – 5

9 a Runge-Kutta method, solve $\frac{d^2y}{dx^2} = x\left(\frac{dy}{dx}\right)^2 - y^2$ for x=0.2 correct to four decimal places, using the initial conditions y=1 and $y' = 0$ when x=0. 7 marks

b Solve the variational problem $\int_0^1 (12xy + y'^2)dx$ under the conditions $y(0) = 3$ and $y(1) = 6$. 7 marks

c A heavy cable hangs freely under gravity between two fixed points. Show that the shape of the cable is a catenary. 6 marks

Or

10 a Apply Milne's method to solve $\frac{d^2y}{dx^2} = 1 + \frac{dy}{dx}$ given the following table of initial values. 7 marks

X	0	0.1	0.2	0.3
Y	1	1.1103	1.2427	1.399
y'	1	1.2103	1.4427	1.699

Compute y (0.4) numerically.

b Derive Euler's equation in the Standard form $\frac{\partial f}{\partial y} - \frac{d}{dx}\left(\frac{\partial f}{\partial y'}\right) = 0$ 7 marks

c Prove that the shortest distance between two points in a plane is a straight line joining them. 6 marks

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